

# PARAMETRIC COST MODELS FOR SPACE TELESCOPES

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## I. INTRODUCTION

Multivariable parametric cost models for space telescopes provide several benefits to designers and space system project managers. They identify major architectural cost drivers and allow high-level design trades. They enable cost-benefit analysis for technology development investment. And, they provide a basis for estimating total project cost. A survey of historical models found that there is no definitive space telescope cost model. In fact, published models vary greatly [1]. Thus, there is a need for parametric space telescopes cost models. An effort is underway to develop single variable [2] and multi-variable [3] parametric space telescope cost models based on the latest available data and applying rigorous analytical techniques.

Specific cost estimating relationships (CERs) have been developed which show that aperture diameter is the primary cost driver for large space telescopes; technology development as a function of time reduces cost at the rate of 50% per 17 years; it costs less per square meter of collecting aperture to build a large telescope than a small telescope; and increasing mass reduces cost.

## II. MODEL CREATION

To develop a parametric cost models requires data. Cost and engineering data has been collected on 59 different parameters for 23 different UV, optical or infrared space telescopes. (Table 1 and 2)

Table 1: UV/OIR Cost Model Missions Database	
UV/Optical Telescopes	Infrared Telescopes
EUVE	CALIPSO
FUSE	Herschel
GALEX	ICESat
HiRISE	IRAS
HST	ISO
HUT	JWST
IUE	SOFIA
Kepler	Spitzer (SIRTF)
Copernicus (OAO-3)	TRACE
SOHO/EIT	WIRE
UIT	WISE
WUPPE	

Statistical correlations have been evaluated between 19 variables and used to develop single and multi-variable cost estimating relationships (CERs) to model Optical Telescope Assembly (OTA) and Total Mission Cost. CERs are evaluated for their 'goodness'.

Optical Telescope Assembly (OTA) is defined as the space observatory subsystem which collects electromagnetic radiation and focuses it (focal) or concentrates it (afocal). An OTA consists of the primary mirror, secondary mirror, auxiliary optics and support structure (such as optical bench or truss structure, primary support structure, secondary support structure or spiders, etc.). An OTA does not include science instruments or spacecraft subsystems. Cost is defined as prime contract cost without any NASA labor or overhead. Total mission cost is defined as Phase A-D cost, excluding: launch cost; costs associated with NASA labor (civil servant or support contractors) for program management, technical insight/oversight; or any NASA provided ground support equipment, e.g. test facilities. Accounting for NASA overheads would increase the cost by at least 10% and maybe as much as 33%.

Table 2: Cost Model Variables Study and the completeness of data knowledge	
Parameters	% of Data
OTA Cost	89%
Total Phase A-D Cost w/o LV	84%
Aperture Diameter	100%
Avg. Input Power	95%
Total Mass	89%
OTA Mass	89%
Spectral Range	100%
Wavelength Diffraction Limit	63%
Primary Mirror Focal Length	79%
Design Life	100%
Data Rate	74%
Launch Date	100%
Year of Development	95%
Technology Readiness Level	47%
Operating Temperature	95%
Field of View	79%
Pointing Accuracy	95%
Orbit	89%
Development Period	95%
Average	88%

Goodness of a Fit or a Correlation is tested via a range of statistical measures, including Pearson's  $r^2$  coefficient, Student T-Test p-value and standard percent error (SPE). Pearson's  $r^2$  (typically denoted as just  $r^2$ ) describes the percentage of agreement between the model and the actual cost. For multi-variable models, we use Adjusted Pearson's  $r^2$  (or  $r^2_{adj}$ ) which accounts for the number of data points and the number of variables. In general, the closer  $r^2$  (or  $r^2_{adj}$ ) is to 1.0 or 100%, the better the model. SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit. The closer SPE is to 0, the better the fit. Please note that since SPE is normalized, a small variation divided by a very small fit value can yield a very large SPE. The p-value is the probability that a fit or correlation would occur if the variables are independent of each other. The closer the p-value is to 0, the more significant the fit or correlation. The closer it is to 1, the less significant. If the p-value for a given variable is small, then removing it from the model would cause a large change to the model. If it is large, then removing the variable will have a negligible effect. Also, it is important to consider how many data points are included in a given correlation or fit.

Table 3 summarizes the cross-correlation between specific key parameters and Total Mission Cost, OTA Cost and OTA Areal Cost (where areal cost is defined as OTA cost divided by OTA collecting area). For each parameter, Table 3 reports its correlation to cost, the correlation's p-value and the number of data points in the correlation. Diameter appears to be the most significant cost driver. So, in addition to total cost and OTA cost we have examined OTA Areal Cost, i.e. OTA Cost per unit Area of Primary Mirror collecting aperture. Diameter is

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Corr	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

with either. As expected, Total Mass correlates most significantly with Total Cost while OTA Mass correlates most significantly with OTA Cost. Unexpectedly, Minimum Spectral Range Value and Operating Temperature do not have a significant correlation with any Cost. However, Spectral Minimum does have a role in multi-variable cost models. As expected Electrical Power, Design Life and Development Period have significant correlations (99% confidence) with Total Cost. Also unexpected is that TRL and Launch Year do not have significant correlations. But, they both have roles in multi-variable cost models. One problem with TRL is that there are only 8 data points. Also, it is a qualitative and not a quantitative parameter.

### III. COST MODELS

Four single variable cost estimating relationships (CERs) have been developed for OTA cost and total mission cost as a function of OTA diameter, OTA mass and total mission mass [2]. These models were developed with and without JWST. The benefit of including JWST is that it is the most current mission. The disadvantage is that its cost is not yet final. For the purpose of this paper, we will include the 2009 JWST C/D final cost estimate. In general, including JWST does affect the model  $r^2_{adj}$  but does not increase the noisiness of the fit as represented by the SPE. Additionally, these models are developed only for free-flying missions. Of the 23 missions in the data base, there are 19 free flying telescopes (17 for which we have OTA cost data) and 4 that are attached (3 to the Space Shuttle Orbiter and SOFIA to a Boeing 747 airplane). As will be discussed below with regard to mass models, attached missions have a significantly different cost dependency than free-flying missions. Therefore, we excluded attached missions from the models.

Engineering judgment says that OTA cost is most closely related to OTA engineering parameters. But, managers and mission planners are more interested in total Phase A-D cost. Analysis of the 14 free-flying missions for which we have both OTA cost data and Phase A-D Total Mission cost data indicates (Fig 1) that OTA cost is ~20% of total mission cost ( $R^2 = 96\%$ ) with a model residual standard deviation of approximately \$300M. It is interesting to note that there is significant variation in this percentage for small missions but not for large. Additionally, we created a common Work Breakdown Structure (WBS) and mapped onto it the individual WBSs of 7 missions (including HST and JWST) for which we had detailed cost data. This analysis indicates that OTA cost is 30% of Total (Fig 2).

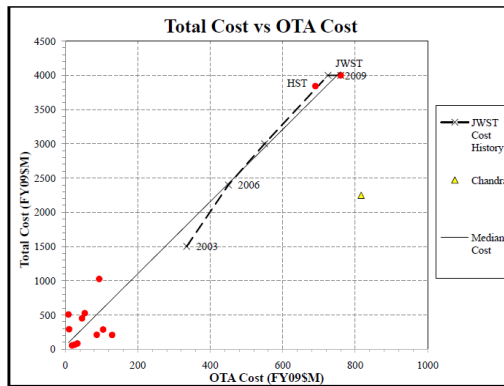


Fig 1: Total Mission Cost vs Percentage that OTA Cost is of Total Cost.

#### Typical Space Telescope Cost Breakdown

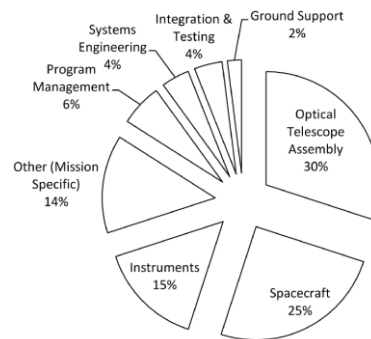


Fig 2: Average WBS cost allocation for 7 free flying UV/OIR systems.

Fig 3 plots OTA Cost for free-flying space telescopes as a function of Primary Mirror Diameter. The regression fit for this data is:

$$\text{OTA Cost} \sim \text{Aperture Diameter}^{1.2} \quad (N = 17; r^2 = 75\%; \text{SPE} = 79\%) \text{ with 2009 JWST}$$

Note that the Chandra data point is for reference only. It is not included in the regression. And, it is plotted based upon the equivalent normal incidence mirror diameter it would have if all of its x-ray mirrors were unrolled.

Given that the OTA cost might be dominated by the large apertures for HST and JWST, a model was also created for normalized Areal OTA Cost (Fig 4):

$$\text{OTA Areal Cost} \sim \text{Aperture Diameter}^{-0.74} \quad (N = 17; r^2 = 55\%; \text{SPE} = 78\%) \text{ with JWST}$$

A key finding of this analysis is that Areal Cost decreases with aperture size. It is less expensive per photon to build a large aperture telescope than a small aperture telescopes. Large aperture telescopes provide a better ROI.

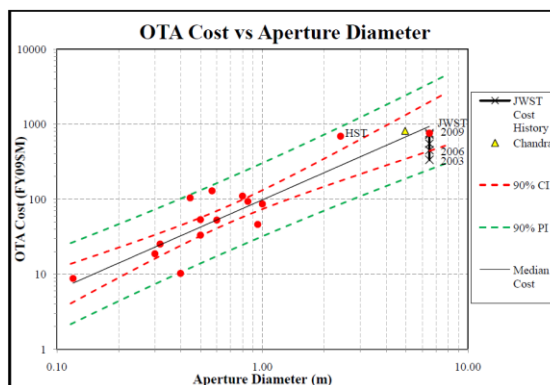


Fig 3: OTA Cost vs Aperture Diameter scaling law for 17 free flying UV/OIR systems (including 2009 JWST). Plot includes 90% confidence and prediction intervals, and data points. Chandra data is not in the regression.

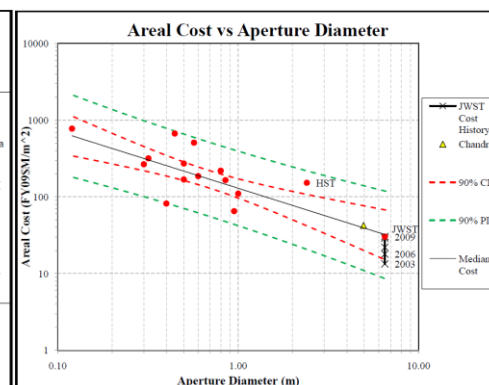


Fig 4: OTA Areal Cost vs Aperture Diameter scaling law for 17 free flying UV/OIR systems (including 2009 JWST). Plot includes 90% confidence and prediction intervals, and data points. Chandra is not in the regression.

From an engineering and a scientific perspective, aperture is the best parameter to build a space telescope cost model. Aperture defines the observatory's science performance and determines the payload's size and mass. And, while the results are consistent with some historical cost models, our results invalidate long held 'intuitions' which are often purported to be 'common knowledge'. Space telescope costs vary almost linearly with diameter and not to a power of 1.6X or 2.0X or even 2.8X. But, a model based on diameter alone has only a ~75% agreement with the OTA cost data and ~55% agreement with the OTA areal data. Therefore, a multi-variable step wise regression is required to look for other factors which influence cost. First, one performs a two variable regression of Diameter plus each of the other parameters and evaluates the statistical 'goodness' of each regression (Fig 5). Once a good two variable model is selected, the process can be repeated to add a third variable.

Second Variable	coef		p		OTA Cost versus Diameter and V2																	
	Diameter alone		PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit		Operating Temperature	
Diameter	1.20	0.00	0.68	0.27	1.05	0.00	-0.02	0.99	1.16	0.01	1.14	0.00	0.76	0.12	1.45	0.00	1.22	0.00	1.19	0.00	1.21	0.00
Second Variable	-	-	0.35	0.45	0.26	0.57	0.35	0.45	-0.26	0.18	-0.05	0.45	0.35	0.26	0.35	0.26	-0.04	0.63	-0.10	0.55	-0.01	0.96
Adjusted r2	73%		71%		71%		71%		14%		73%		83%		83%		73%		75%		71%	
SPE	79%		77%		78%		77%		73%		78%		83%		83%		84%		95%		82%	
n	17		13		13		13		13		16		15		15		17		11		16	
Multicollinearity?	N/A		Yes		No		Yes		No		No		Yes		No		No		No		No	

Second Variable			Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		YoD (exp)		Development Period		Dev Per (exp)		Launch Date (exp)		Orbit	
Diameter	1.41	0.00	1.40	0.00	1.21	0.00	1.13	0.00	1.31	0.00	1.27	0.00	1.19	0.00	1.20	0.00	1.34	0.00	1.23	0.00		
Second Variable	-0.15	0.23	-0.08	0.28	-0.01	0.98	0.00	0.51	-0.09	0.02	-0.04	0.00	0.23	0.60	0.00	0.73	-0.04	0.00	0.02	0.62		
Adjusted r2	70%		91%		71%		84%		97%		95%		71%		71%		93%		66%			
SPE	58%		59%		83%		81%		83%		39%		77%		78%		39%		85%			
n	16		12		17		17		8		16		16		16		17		15			
Multicollinearity?	No		No		No		No		No		No		No		No		No		No			

Fig 5: Two variable regression for OTA Cost vs Aperture Diameter and a 2<sup>nd</sup> Variable

Regarding potential two variable OTA cost models, three parameters have significance greater than 98%: TRL, Year of Development (YoD) and Launch Year (LYr). The Diameter + TRL model has a slightly higher  $r^2_{adj}$  than the other models, but it also has a high SPE. This may be because of the relatively few TRL data points in our data base. Or, it may be because TRL value is subjective and thus has a natural 'fuzziness' to its data values. Based on coefficient significance, other parameters of potential interest are Field of View (82%), OTA Mass (74%), OTA Areal Density (74%), Power (77%) and Data Rate (72%). But, all, except Data Rate, do not simultaneously increase  $r^2_{adj}$  and decrease SPE. And, some, such as FOV, are particularly poor. It should also be noted that OTA Mass is multicollinear with Aperture Diameter – which only makes sense, i.e. the larger the telescope, the more mass it should have. Therefore, mass is not a good second variable candidate.

Both YoD and LYr have similarly high  $r^2_{adj}$  values and significantly lower SPE values. And, if you round significant digits, each model is virtually identical:

$$\text{OTA Cost} \sim D^{1.34} e^{-0.04(\text{LYr}-1960)} \quad (N = 17, r^2_{adj} = 93\%; \text{SPE}=39\%)$$

$$\text{OTA Cost} \sim D^{1.27} e^{-0.04(\text{YoD}-1960)} \quad (N = 16, r^2_{adj} = 95\%; \text{SPE}=39\%)$$

Launch Year has the advantage of being a definite date, but it has the disadvantage that a launch can be delayed. However, while a launch delay tends to increase the total mission cost, it may not increase OTA cost. Year of Development yields a slightly better regression, but its exact date is subject to definition. Does it start with Phase A or Phase C? Regardless of which parameter is used, the message is clear: technology improvements reduce OTA cost as a function of time by approximately 50% every 17 years. For completeness, a two variable OTA Areal Cost regression yielded the same basic results.

The next step is to try adding a third parameter. For our data base of free-flying missions, two different regressions were performed for OTA Cost versus Diameter, a 'year' parameter and each of the other variables as the third parameter. Neither regression yielded a satisfactory model. Next, we decided to add some wavelength diversity by

including missions with shorter and longer wavelengths. Specifically, we added WMAP, TDRS-1, TDRS-7, EUVE, Chandra and Einstein. With the extra missions, two satisfactory three variable model was achieved:

$$\text{OTA Cost} \sim D^{1.15} \lambda^{-0.17} e^{-0.03(YoD-1960)} \quad (N = 20, r^2_{adj} = 92\%; \text{SPE} = 76\%)$$

$$\text{OTA Cost} \sim D^{1.05} \lambda^{-0.13} e^{-0.03(LY-1960)} \quad (N = 23, r^2_{adj} = 63\%; \text{SPE} = 69\%)$$

Finally, while aperture is the single most important parameter driving science performance, system mass determines what vehicle can be used to launch it. Also, significant engineering costs are expended to keep a given payload inside of its allocated mass budget, including light-weighting mirrors and structure. Therefore, mass is a potential important CER.

Fig 6 plots Total Cost vs Total Mission Mass for 15 free-flying missions. The regression of this data is:

$$\text{Total Cost} \sim \text{Total Mass}^{1.12} \quad (N = 15; r^2 = 86\%; \text{SPE} = 71\%) \text{ with JWST}$$

Fig 7 plots OTA Cost vs OTA Mass for both free-flying and attached missions. The regression for only the free-flying missions is:

$$\text{OTA Cost} \sim \text{OTA Mass}^{0.72} \quad (N = 15; r^2 = 92\%; \text{SPE} = 93\%) \text{ with JWST}$$

While OTA Mass may appear to be a good indicator of OTA Cost because it has the highest Pearson's  $r^2$ , it also has the highest SPE. And, please note that just because we have created a mass CER, we do not recommend using it. In general mass should be avoided as a CER because it is a secondary indicator. Mass depends upon the size of the telescope. Bigger telescopes have more mass. And, bigger telescopes require bigger spacecraft and bigger science instruments which require more power – all which adds mass. And, because many missions are designed to a mass-budget defined by launch vehicle constraints, the result can be a very complex, risky, and expensive mission architecture when trying to extend the state-of-the-art in either wavelength or aperture. This effect can be seen in Fig 6 where JWST has nearly half the total mass of HST but still has a higher total mission cost – because JWST is bigger and more complex than HST. But, this does not have to be the case.

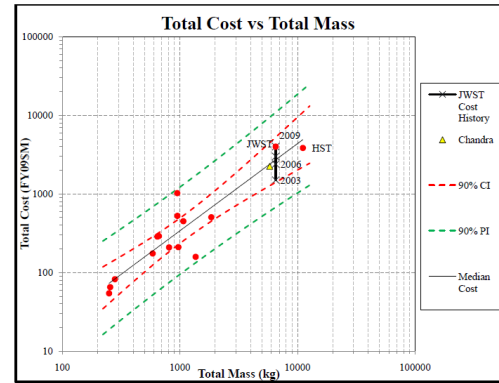


Fig 6: Free-Flying Total Cost vs Mass

As indicated in Fig 7 and Fig 8, it is possible to reduce cost by building space telescopes with different design rules. Fig 7 shows that Attached OTAs have a different cost versus mass relationship than free-flying OTAs. The reason is that ‘attached’ OTAs have a much more relaxed mass budget constraint than ‘free-flying’ OTAs. Fig 8 shows two key findings. First, the OTA cost per kilogram is entirely different for free-flying versus attached missions. Attached OTAs are approximately 5.5X less expensive per kg than free-flying OTAs. Second, the cost per kg for these classes of missions is independent of aperture size. Other analysis shows that for a given aperture size, attached OTAs are on average ~2X more massive and ~2.5X less expensive than free-flying OTAs. Finally, there may be a third cost class – Planetary – but we are not certain because HiRISE is our only planetary OTA data point.

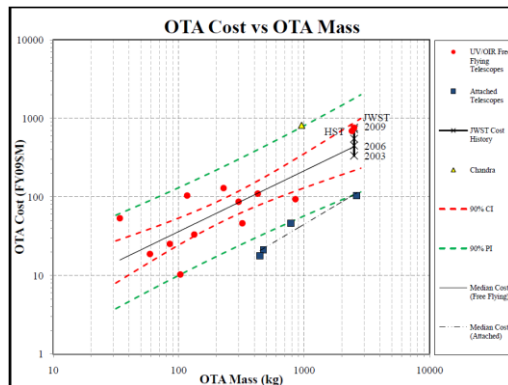


Fig 7: OTA Cost vs OTA Mass

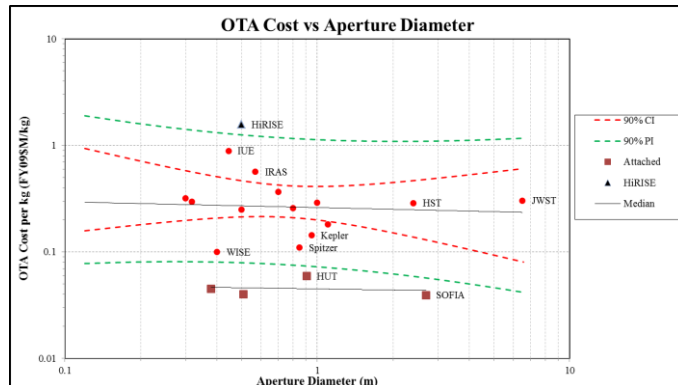


Fig 8: OTA Cost per kilogram vs OTA Aperture Diameter

The importance of these findings is that they invalidates the ‘common assumption’ that the more massive the mission the more expensive the mission. The only reason that more massive missions are more expensive is because they have more ‘stuff’. When one compares missions with similar performance properties, it is less expensive to design, build and fly a simple mission with more mass than a lightweight complex mission. Therefore, maybe the best way to reduce the cost of future large aperture space telescopes is to develop cost effective heavy lift launch vehicles which will enable mission planners to trade complexity for mass.

#### IV. CONCLUSIONS

Cost models are invaluable for system designers. They identify major architectural cost drivers and allow high-level design trades. They enable cost-benefit analysis for technology development investment. And, they provide a basis for estimating total project cost. A study is in-process to develop single and multivariable parametric cost model for space telescopes. Cost and engineering parametric data has been collected on 30 different missions and extensively analyzed for 23 normal incidence UV/OIR space telescopes. Statistical correlations have been developed for 19 of the 59 variables sampled.

From an engineering & science perspective, Aperture Diameter is the best parameter for a space telescope cost model. But, the single variable model only predicts 75% of OTA Cost:

$$\text{OTA Cost} \sim D^{1.2} \quad (N = 17; r^2_{\text{adj}} = 75\%; \text{SPE}=79\%) \text{ with 2009 JWST}$$

Two and three variable models provide better estimates:

$$\text{OTA Cost} \sim D^{1.3} e^{-0.04(\text{LYr}-1960)} \quad (N = 17, r^2_{\text{adj}} = 93\%; \text{SPE}=39\%)$$

$$\text{OTA Cost} \sim D^{1.3} e^{-0.04(\text{YoD}-1960)} \quad (N = 16, r^2_{\text{adj}} = 95\%; \text{SPE}=39\%)$$

$$\text{OTA Cost} \sim D^{1.15} \lambda^{-0.17} e^{-0.03(\text{YoD}-1960)} \quad (N = 20, r^2_{\text{adj}} = 92\%; \text{SPE} = 76\%)$$

where: D = Aperture Dia, LYr = Launch Yr, YoD = Yr of Development, and  $\lambda$  = Spectral Min Wavelength.

At present the study has not yet produced a satisfactory model for Total Mission Cost.

While mass does yield a statistically significant regression which implies that more massive telescopes cost more, this finding is artificial, misleading, could easily lead one to make inappropriate programmatic decisions, and it contradicts the fact that JWST costs more than HST but has half the mass. A careful study of the data actually indicates that for any given aperture diameter, attached OTAs are on average 2X more mass and 2.5X less expensive than free-flying OTAs; the cost per kilogram of attached OTAs is ~5.5X lower than for free-flying OTAs; and that the cost per kg of these two ‘design rule’ classes is independent of aperture. Finally, there may be a third even more expensive ‘design rule’ class – Planetary OTAs – but we only have one data point currently in the data base.

The primary conclusions of the cost modeling study to date are:

- The primary cost driver for Space Telescope Assemblies is Aperture Diameter.
- It costs less per collecting area to build a large aperture telescope than a small aperture telescope.
- Technology development as a function of time reduces cost at the rate of 50% per 17 years.
- If all other parameters are held constant, adding mass reduces cost and reducing mass increases cost.

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# Parametric Cost Models for Space Telescopes

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# Parametric Cost Models

Parametric cost models have several uses:

- high level mission concept design studies,
- identify major architectural cost drivers,
- allow high-level design trades,
- enable cost-benefit analysis for technology development investment, and
- provide a basis for estimating total project cost.





# Data Collection Methodology



# Methodology

Data on 59 different variables was acquired for 30 NASA, ESA, & commercial space telescopes using:

- NAFCOM (NASA/ Air Force Cost Model) database,
- RSIC (Redstone Scientific Information Center),
- REDSTAR (Resource Data Storage and Retrieval System),
- project websites, and interviews.

Table 1: Cost Model Missions Database	
<u>X-Ray Telescopes</u> Chandra (AXAF) Einstein (HEAO-2)	<u>Infrared Telescopes</u> CALIPSO Herschel ICESat IRAS ISO JWST SOFIA Spitzer (SIRTF) TRACE WIRE WISE
<u>UV/Optical Telescopes</u> EUVE FUSE GALEX HiRISE HST HUT IUE Kepler Copernicus (OAO-3) SOHO/EIT UIT WUPPE	<u>Microwave Telescopes</u> WMAP
	<u>Radio Wave Antenna</u> TDRS-1 TDRS-7



# Missions

Of the 30 mission, we initially studied 23 ‘normal-incidence’ UVOIR and Infrared telescopes.

Of these,

19 are ‘Free Flying’

4 are ‘Attached’, and

1 is ‘Planetary’

For wavelength diversity, added microwave, radio wave and grazing incidence X-Ray/EUV.

Table 1: Cost Model Missions Database	
<u>X-Ray Telescopes</u> Chandra (AXAF) Einstein (HEAO-2)	<u>Infrared Telescopes</u> CALIPSO Herschel ICESat IRAS ISO JWST SOFIA Spitzer (SIRTF) TRACE WIRE WISE
<u>UV/Optical Telescopes</u> EUVE FUSE GALEX HiRISE HST HUT IUE Kepler Copernicus (OAO-3) SOHO/EIT UIT WUPPE	<u>Microwave Telescopes</u> WMAP
	<u>Radio Wave Antenna</u> TDRS-1 TDRS-7



# Cost Variables

Total Cost is Phase A through D, it does not include:

- Phase E (post-launch) costs
- Launch related costs
- Civil servant costs (NASA employees)
- So our Total Cost is contract cost to make the system.

OTA Cost includes only:

- Primary mirror
- Secondary (and tertiary if appropriate) mirror(s)
- Related support structure

Total Mass and OTA Mass match the cost definitions



# Technical Variables

Aperture Diameter

Mass (OTA and Total)

PM Focal Length

PM F/#

Field of View

Pointing Accuracy

Spectral Range Minimum

Wavelength of Diffraction Limit

Operating Temperature

Average Input Power

Data Rate

Design Life

Orbit



# Programmatic Variables

Launch Year

Year of Development (or Start of Development)

Development Period

TRL (Technology Readiness Level)



# Completeness of Data for 19 Variables

Table 2: Cost Model Variables Study and the completeness of data knowledge	
Parameters	% of Data
OTA Cost	89%
Total Phase A-D Cost w/o LV	84%
Aperture Diameter	100%
Avg. Input Power	95%
Total Mass	89%
OTA Mass	89%
Spectral Range	100%
Wavelength Diffraction Limit	63%
Primary Mirror Focal Length	79%
Design Life	100%
Data Rate	74%
Launch Date	100%
Year of Development	95%
Technology Readiness Level	47%
Operating Temperature	95%
Field of View	79%
Pointing Accuracy	95%
Orbit	89%
Development Period	95%
Average	88%



# Statistical Analysis Methodology





# Model Creation

Start with Correlation Matrix.

Look for Variables which are Highly Correlated with Cost.

The higher the correlation the greater the Cost Variation which is explained by a given Variable.

Sign of correlation is important and must be consistent with Engineering Judgment.

Important for Multi-Variable Models:

We want Variables which Independently effect Cost.

When Variables 'cross-talk' with each other it is called Multi-Collinearity.

Thus, avoid Variables which are highly correlated with each other.



# Goodness of Correlation, Fits and Regressions

‘Correlation’ between variables and ‘Goodness’ of single variable models is evaluated via Pearson’s  $r^2$  standard percent error (SPE), and Student’s T-Test p-value.

‘Goodness’ of multivariable fits are evaluated via Pearson’s Adjusted  $r^2$  which accounts for number of data points and number of variables.

Pearson’s  $r^2$  coefficient describes the percentage of agreement between the fitted values and the actual data.

The closer  $r^2$  is to 1, the better the fit.

SPE is a normalized standard deviation of the fit residual (difference between data and fit) to the fit.

The closer SPE is to 0, the better the fit



# Significance

The final issue is whether or not a correlation or fit is significant.

p-value is the probability that the fit or correlation would occur if the variables are independent of each other.

The closer p-value is to 0, the more significant the fit or correlation.

The closer p-value is to 1, the less significant.

If the p-value for a given variable is small, then removing it from the model would cause a large change to the model.

If p-value is large, then removing the variable will have a negligible effect

It is only possible to 'test' if the correlation between two variables is significant.

It is not possible to 'test' if two variables are independent.



# Cross Correlation Matrix

	Total Phase A-D Cost	OTA Cost	Areal OTA Cost	Aperture Diameter	PM F Len.	PM F/N	OTA Volume	FOV	Pointing Accuracy	Total Mass	OTA Mass	OTA Areal Density	Spectral Range minimum	Diffraction Limit	Operating Temperature	Avg. Input Power	Data Rate	Design Life	Technology Readiness Level	Year of Development	Development Period	Launch Date	Orbit
units	(FY09\$M)	(FY09\$M)	(FY09\$M/m <sup>2</sup> )	(m)	(m)	unitless	(m <sup>3</sup> )	(°)	(Arc-Sec)	(kg)	(kg)	(kg/m <sup>2</sup> )	(μ)	(μ)	(K)	(Watts)	(Kbps)	(months)	TRL	(year)	(months)	(year)	(km)
Total Phase A-D Cost	<b>1.00</b>	<b>0.70</b>	-0.36	<b>0.64</b>	<b>0.80</b>	0.38	<b>0.83</b>	0.26	-0.52	<b>0.92</b>	<b>0.72</b>	-0.48	-0.02	-0.40	-0.04	0.59	0.44	<b>0.65</b>	-0.41	-0.11	<b>0.78</b>	0.11	0.54
OTA Cost		<b>1.00</b>	-0.30	<b>0.87</b>	<b>0.82</b>	0.39	<b>0.84</b>	0.00	-0.58	<b>0.68</b>	<b>0.82</b>	-0.41	0.07	-0.23	0.01	0.14	0.15	0.46	<b>-0.68</b>	-0.31	0.45	-0.16	0.17
Areal OTA Cost			<b>1.00</b>	<b>-0.74</b>	<b>-0.62</b>	-0.16	<b>-0.71</b>	-0.56	0.30	-0.34	-0.48	0.59	-0.20	-0.07	-0.03	-0.48	-0.48	-0.41	-0.43	-0.56	-0.22	<b>-0.68</b>	0.04
Aperture Diameter				<b>1.00</b>	<b>0.88</b>	0.27	<b>0.98</b>	-0.09	-0.58	<b>0.63</b>	<b>0.86</b>	<b>-0.60</b>	0.14	-0.11	0.05	0.42	0.38	0.53	-0.29	0.09	0.37	0.26	0.08
PM F Len.					<b>1.00</b>	<b>0.69</b>	<b>0.96</b>	0.34	<b>-0.66</b>	<b>0.84</b>	<b>0.78</b>	-0.44	-0.50	-0.19	0.28	0.49	0.31	0.50	-0.38	-0.07	0.50	0.10	0.28
PM F/N						<b>1.00</b>	0.45	0.57	-0.41	0.48	0.33	-0.02	<b>-0.61</b>	-0.43	0.32	0.06	0.20	0.25	-0.37	-0.32	0.21	-0.29	0.08
OTA Volume							<b>1.00</b>	0.08	<b>-0.65</b>	<b>0.84</b>	<b>0.84</b>	-0.54	-0.36	-0.08	0.21	<b>0.65</b>	0.34	0.52	-0.31	0.06	0.54	0.26	0.31
FOV								<b>1.00</b>	0.12	0.16	-0.05	0.01	0.05	-0.38	-0.06	-0.02	0.18	0.09	-0.27	0.08	-0.01	0.09	0.09
Pointing Accuracy									<b>1.00</b>	-0.48	<b>-0.71</b>	0.14	0.31	0.08	-0.38	-0.37	-0.29	-0.35	-0.15	0.13	-0.55	-0.02	-0.32
Total Mass										<b>1.00</b>	<b>0.82</b>	-0.42	-0.15	-0.49	0.03	0.55	0.17	<b>0.65</b>	-0.56	-0.27	<b>0.64</b>	-0.10	0.33
OTA Mass											<b>1.00</b>	-0.11	-0.06	0.06	-0.03	0.60	0.09	0.40	-0.29	-0.16	0.57	0.02	0.47
OTA Areal Density												<b>1.00</b>	0.05	0.28	-0.31	-0.16	-0.39	-0.55	0.07	-0.36	-0.20	-0.46	-0.09
Spectral Range minimum													<b>1.00</b>	<b>0.76</b>	<b>-0.79</b>	-0.09	-0.12	-0.25	-0.09	0.21	0.20	0.23	0.01
Diffraction Limit														<b>1.00</b>	-0.55	-0.07	-0.25	<b>-0.75</b>	0.51	0.35	0.31	0.28	0.14
Operating Temperature															<b>1.00</b>	0.09	0.31	0.31	0.11	-0.01	-0.30	0.00	-0.30
Avg. Input Power																<b>1.00</b>	0.34	<b>0.64</b>	0.05	0.35	0.27	0.45	0.04
Data Rate																	<b>1.00</b>	0.54	0.51	0.49	-0.06	0.52	0.21
Design Life																		<b>1.00</b>	-0.15	0.12	0.12	0.24	0.14
Technology Readiness																			<b>1.00</b>	<b>0.68</b>	-0.24	<b>0.64</b>	0.33
Year of Development																				<b>1.00</b>	-0.23	<b>0.97</b>	-0.05
Development Period																					<b>1.00</b>	-0.02	0.51
Launch Date																						<b>1.00</b>	0.04
Orbit																							<b>1.00</b>

Correlations which are at least 95% significant are **Bolded**, e.g. for 12 data points a correlation of greater than 60% is significant to better than 95%.



# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Diameter appears to be the most significant cost driver. So, in addition to total cost and OTA cost we have examined OTA Areal Cost, i.e. OTA Cost per unit Area of Primary Mirror collecting aperture. Diameter is correlated with all three with a significance of greater than 99%.



# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Primary Mirror Focal Length is also a significant correlation, but as we will discover later, it is multi-collinear with Diameter. The assumed explanation is that all space telescopes tend to have the same basic PM F/#.



# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
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Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Pointing Accuracy has reasonable correlation with cost. And, as expected from engineering judgment, it has significant correlation (99% confidence level) with diameter and OTA mass. Interesting, as will be discussed later, pointing is not multi-collinear with either.



# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

As expected, Total Mass correlates most significantly with Total Cost while OTA Mass correlates most significantly with OTA Cost.





# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Unexpectedly, Minimum Spectral Range Value and Operating Temperature do not have a significant correlation with any Cost. However, as we will show later, Spectral Minimum does have a role in multi-variable cost models.



# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

As expected Electrical Power, Design Life and Development Period have significant correlations (99% confidence) with Total Cost.



# Correlation Significance Details

Parameter	Total Cost			OTA Cost			OTA Areal Cost		
	Cor	p	N	Corr	p	N	Corr	p	N
Diameter	.68	.007	14	.87	0	16	-.71	.005	14
Focal Length	.82	.002	11	.82	.001	12	-.42	.194	11
Pointing Accuracy	-.53	.061	14	-.64	.011	15	.47	.087	14
Total Mass	.92	0	15	.68	.005	15	-0	.997	15
OTA Mass	.72	.002	15	.82	0	15	-.47	.074	15
Spectral Min	-.02	.934	16	.07	.804	17	-.23	.383	16
Operating Temp	-.04	.884	16	0	.975	16	-.07	.802	16
Electrical Power	.59	.021	15	.14	.611	16	-.05	.862	16
Design Life	.65	.007	16	.46	.064	17	-.20	.454	16
TRL	-.41	.307	8	-.68	.061	8	-.29	.481	8
Development Period	.78	.001	15	.45	.083	15	.14	.830	15
Launch Year	.11	.675	16	-.16	.533	17	-.34	.204	16

Also unexpected is that TRL and Launch Year do not have significant correlations. But, as we will discuss later, they both have roles in multi-variable cost models. One problem with TRL is there are only 8 data points.



# Cost Models

OTA Cost vs Total Mission Cost

OTA Aperture Cost Models

OTA Mass Cost Models



## OTA Cost vs Total Mission Cost

OTA Cost is typically 20% to 30% of Total  
Mission Cost



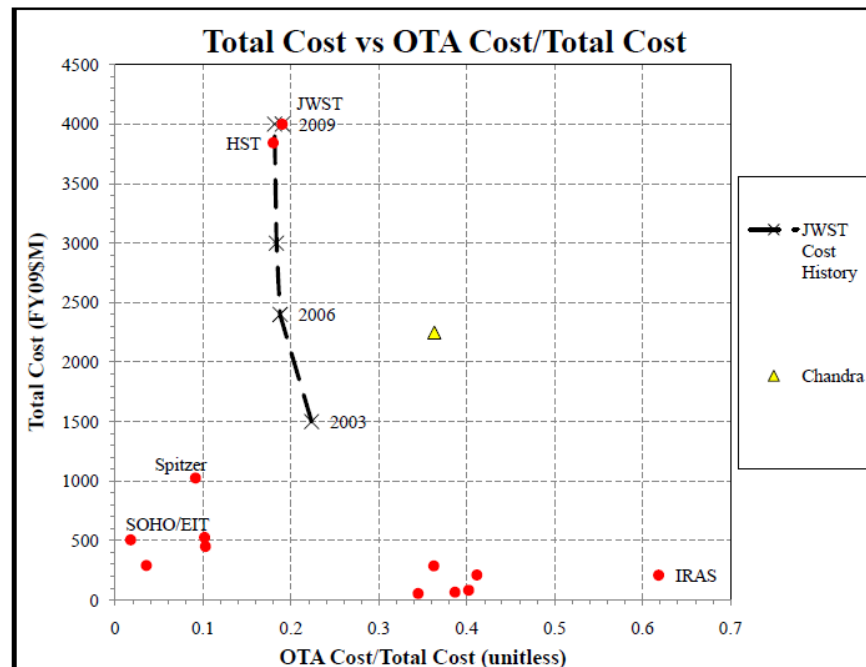
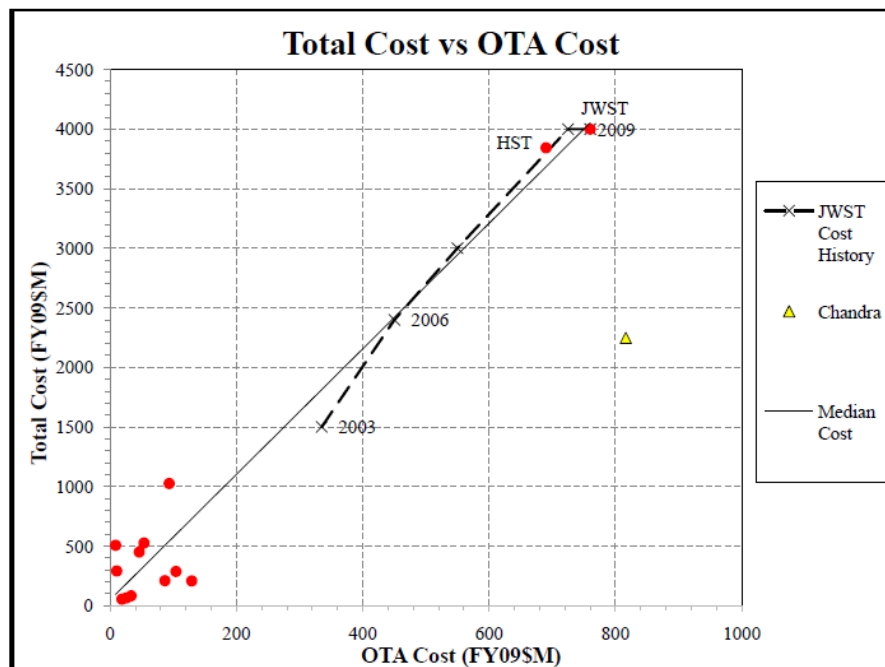
# OTA Cost or Total Cost

Engineering judgment says that OTA cost is most closely related to OTA engineering parameters. But, managers and mission planners are really more interested in total Phase A-D cost.

For 14 missions free flying missions,

OTA cost is ~20% of Phase A-D total cost ( $R^2 = 96\%$ )

with a model residual standard deviation of approximately \$300M.

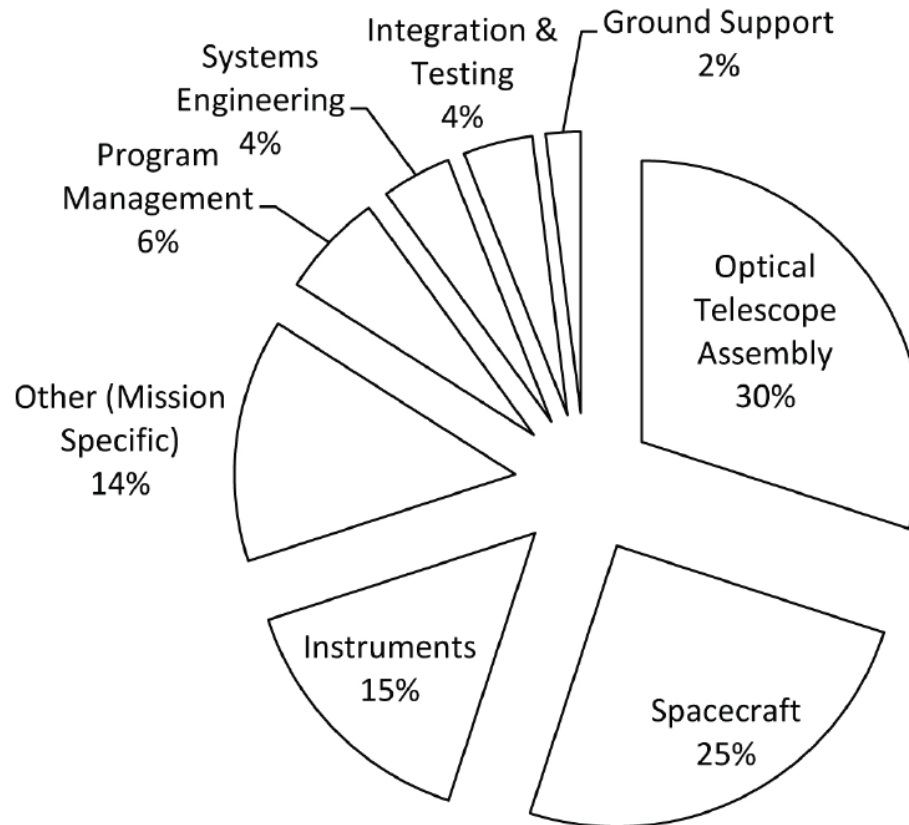




# OTA Cost or Total Cost

We have detailed WBS data for 7 of the 14 free flying missions.  
Mapping on common WBS indicates that OTA is ~30% of Total,

## Typical Space Telescope Cost Breakdown





## Aperture Models

From both an Engineering and a Scientific Perspective, Aperture Diameter is the best parameter for estimating space telescope cost.



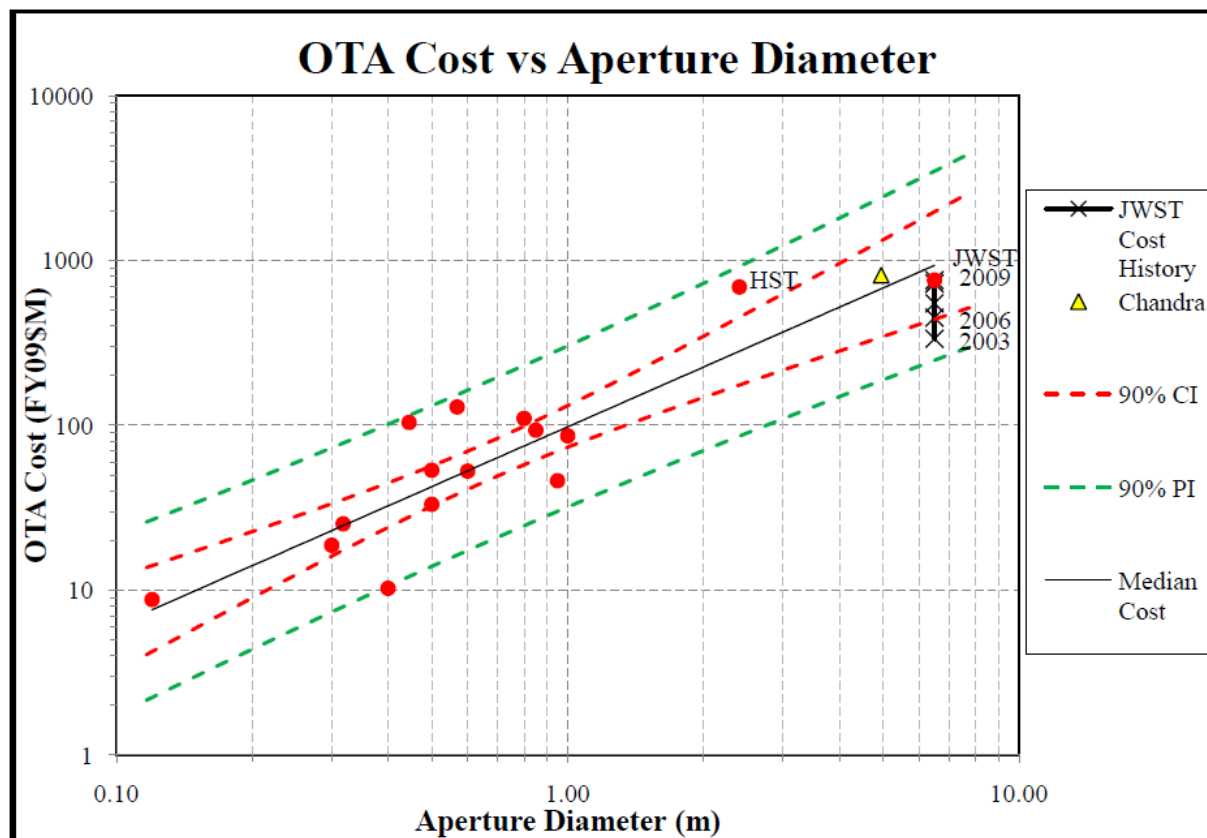


# OTA Cost vs Aperture Diameter

For free-flying space telescopes:

**OTA Cost  $\sim$  Aperture Diameter<sup>1.28</sup>** (N = 16; r<sup>2</sup> = 84%) without JWST

**OTA Cost  $\sim$  Aperture Diameter<sup>1.2</sup>** (N = 17; r<sup>2</sup> = 75%) with 2009 JWST



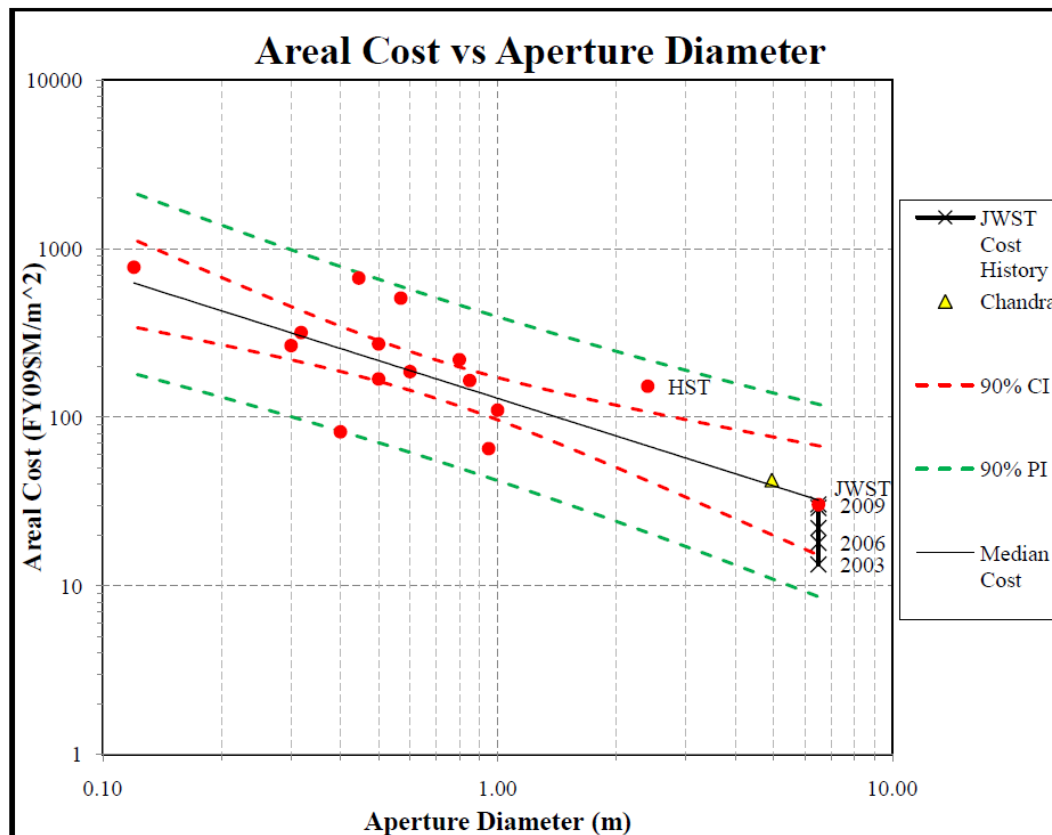


# Area Cost

Total Cost is important, but Areal Cost might be more relevant.

Areal Cost decreases with aperture size, therefore, larger telescopes provide a better ROI

**OTA Areal Cost  $\sim$  Aperture Diameter  $^{-0.74}$**  ( $N = 17$ ;  $r^2 = 55\%$ ) *with JWST*





# Multi-Variable Models

Aperture only models ~75% of OTA Cost variation:

**OTA Cost ~ Aperture Diameter<sup>1.2</sup>** (N = 17;  $r^2 = 75\%$ ) with 2009 JWST

therefore, other factors must also influence cost.

To find these factors, requires multi-variable regression

Select second (or third) factor based on:

- Change in Significance of Diameter to Fit

- Significance of Variable #2 to Fit

- Increase in  $r^2_{\text{adj}}$

- Decrease in SPE

- Multi-Collinearity

Some variables may increase  $r^2_{\text{adj}}$  and/or decrease SPE, but they are not significant or their coefficients are not consistent with engineering judgment or they are multi-collinear.



# OTA Cost versus Diameter and V2

	coef		p		OTA Cost versus Diameter and V2																	
Second Variable	Diameter alone		PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit		Operating Temperature	
Diameter	1.20	0.00	0.68	0.27	1.05	0.00	-0.02	0.99	1.16	0.01	1.14	0.00	0.76	0.12	1.45	0.00	1.22	0.00	1.19	0.00	1.21	0.00
Second Variable	-	-	0.35	0.45	0.26	0.57	0.35	0.45	-0.26	0.18	-0.05	0.45	0.35	0.26	0.35	0.26	-0.04	0.63	-0.10	0.55	-0.01	0.96
Adjusted r2	73%		71%		71%		71%		14%		73%		83%		83%		73%		75%		71%	
SPE	79%		77%		78%		77%		73%		78%		83%		83%		84%		95%		82%	
n	17		13		13		13		13		16		15		15		17		11		16	
Multicollinearity?	N/A		Yes		No		Yes		No		No		Yes		No		No		No		No	
Second Variable			Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		YoD (exp)		Development Period		Dev Per (exp)		Launch Date (exp)		Orbit	
Diameter			1.41	0.00	1.40	0.00	1.21	0.00	1.13	0.00	1.31	0.00	1.27	0.00	1.19	0.00	1.20	0.00	1.34	0.00	1.23	0.00
Second Variable			-0.15	0.23	-0.08	0.28	-0.01	0.98	0.00	0.51	-0.09	0.02	-0.04	0.00	0.23	0.60	0.00	0.73	-0.04	0.00	0.02	0.62
Adjusted r2			70%		91%		71%		84%		97%		95%		71%		71%		93%		66%	
SPE			58%		59%		83%		81%		83%		39%		77%		78%		39%		85%	
n			16		12		17		17		8		16		16		16		17		15	
Multicollinearity?			No		No		No		No		No		No		No		No		No		No	

TRL is 98% significant but is noisy. YoD and LYr provide equal results.



## Two Variable Models

Two second variables best meet all the criteria:

Year of Development (YoD), and

Launch Year (LYr)

Launch Year has the advantage that it is a definite date, but it also has the disadvantage that a launch can be delayed. And, while a launch delay tends to increase the total mission cost, it may or may not increase the OTA cost.

Year of Development yields a slightly better regression, but its exact date is subject to definition. Is it the Start of Phase A or B or C?

To first order, both YoD and Lyr yield the exact same model:

$$\text{OTA Cost} \sim D^{1.3} e^{-0.04(\text{LYr}-1960)} \quad (N = 17, r^2 = 93\%; \text{SPE}=39\%)$$

$$\text{OTA Cost} \sim D^{1.3} e^{-0.04(\text{YoD}-1960)} \quad (N = 16, r^2 = 95\%; \text{SPE}=39\%)$$

Two Variable Model estimates ~95% of Cost Variation

$$\text{OTACost} = \$332M * \text{Diameter}^{1.27} * e^{-0.038*(\text{YoD}-1960)}$$



## Three Variable Models

Three Variable Regression (with the existing data base) did not find any model better than either of the two variable models.

Interestingly, no 'significant' dependency on wavelength or operating temperature was found for OTA or Mission cost.

Potential reasons are:

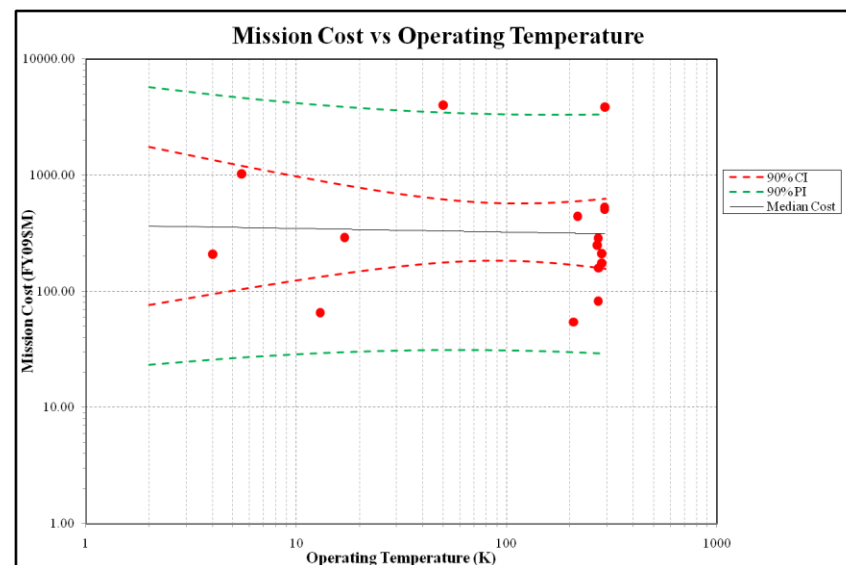
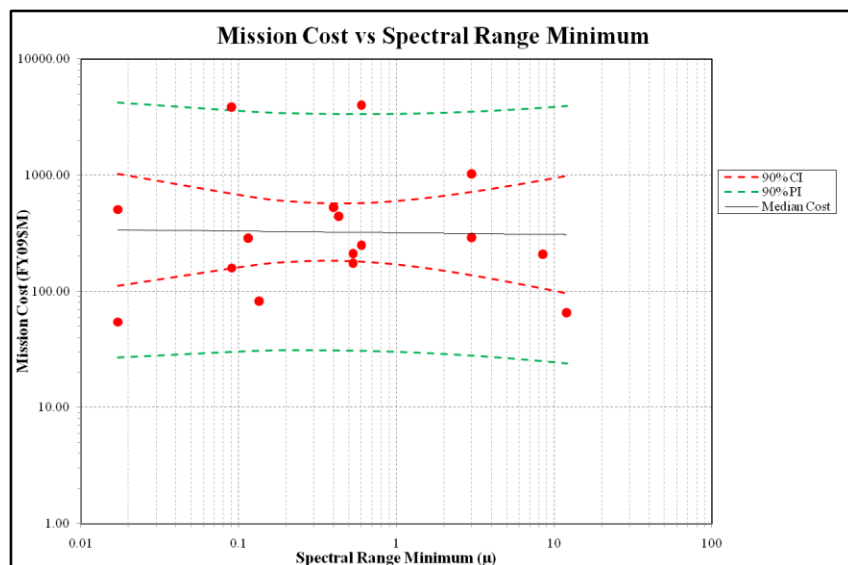
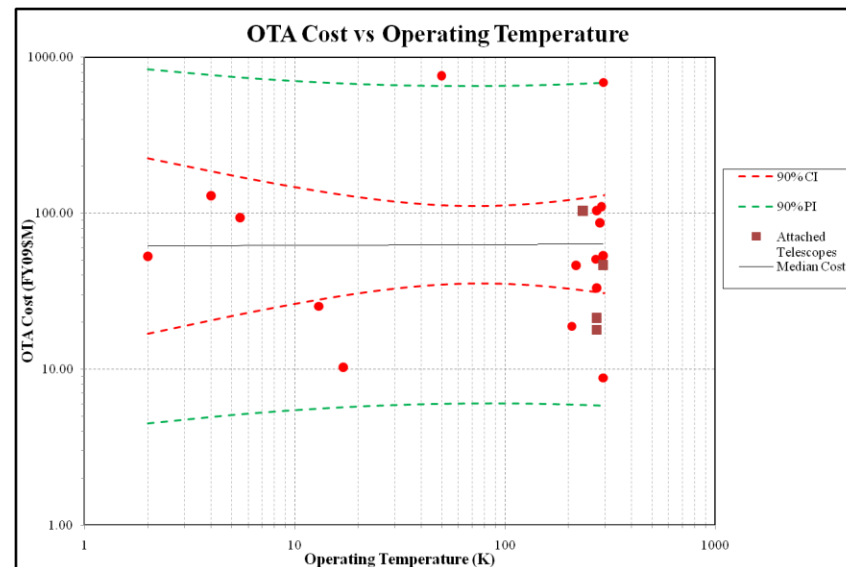
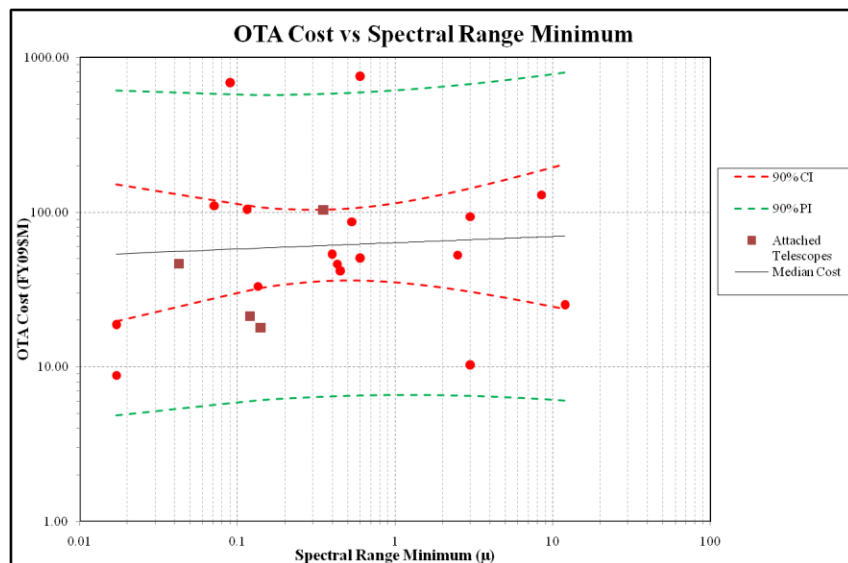
- the difficulty of making a better short wavelength telescope is offset by the ease of ambient operation; and

- the ease of making an infrared telescope is offset by the difficulty of cryogenic operation.



# PRELIMINARY RESULTS

## OTA & Mission Cost vs Wavelength & Temperature





# Three Variable Models

So, we add more high and low wavelength telescopes to gain some wavelength diversity

WMAP, TDRS-1, TDRS-7, EUVE, Chandra and Einstein

	coef	p	OTA Cost vs Diameter, YoD, DoL, and Spct min									
	Diam		Diam, spct min		Diam, YoD(exp)		Diam, YoD(exp), spct min		Diam, DoL(exp)		Diam, DoL(exp), spct min	
Aperture Diameter	0.84	0.00	1.03	0.00	0.78	0.00	1.15	0.00	0.85	0.00	1.05	0.00
YoD	-	-	-	-	-0.03	0.12	-0.03	0.04	-	-	-	-
Launch Date	-	-	-	-	-	-	-	-	-0.03	0.08	-0.03	0.03
Spct Min	-	-	-0.13	0.00	-	-	-0.17	0.00	-	-	-0.13	0.00
Adjusted r2	43%		69%		18%		92%		18%		63%	
SPE	126%		88%		97%		76%		99%		69%	
n	23		23		20		20		23		23	
Multicollinearity?	N/A		No		No		No		No		No	

This resulted in:

$$\text{OTA Cost} \sim D^{1.15} \lambda^{-0.17} e^{-0.03(\text{YoD}-1960)} \quad (N = 20, r^2 = 92\%; \text{SPE} = 76\%)$$





## Mass Models

Be Very Careful – They can be Misleading



# Mass Models

While aperture diameter is the single most important parameter driving science performance.

Total system mass determines what vehicle can be used to launch.

Significant engineering costs are expended to keep a given payload inside of its allocated mass budget.

Such as light-weighting mirrors and structure.

Space telescopes are designed to mass

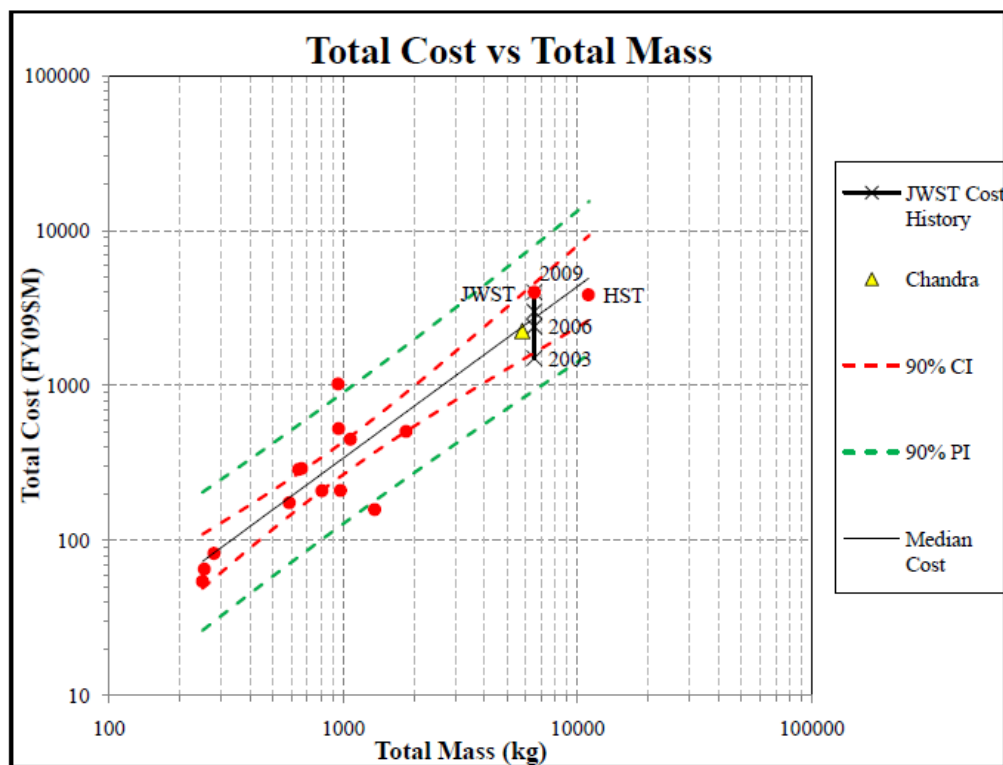


# Total Cost vs Total Mass

Based on 15 free-flying OTAs

**Total Cost ~ Total Mass<sup>1.12</sup>** ( $N = 15$ ;  $r^2 = 86\%$ ) *with JWST*

**Total Cost ~ Total Mass<sup>1.04</sup>** ( $N = 14$ ;  $r^2 = 95\%$ ) *without JWST*



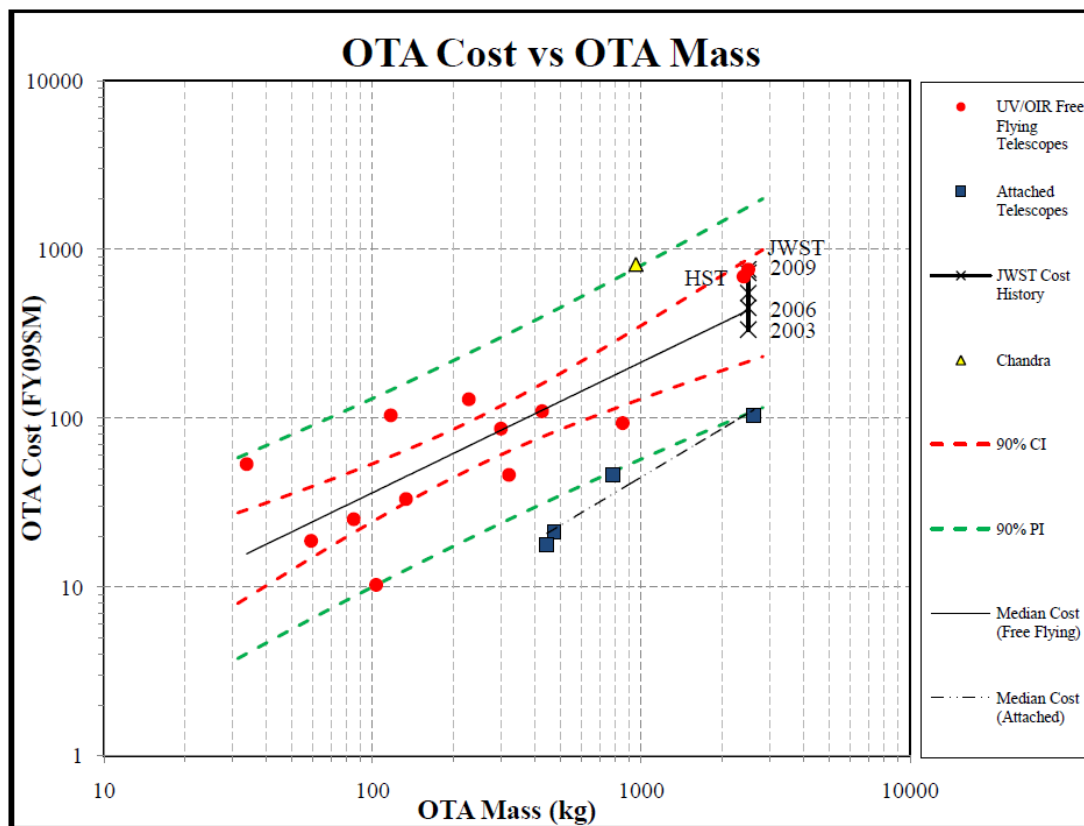


# OTA Cost vs OTA Mass

Based on 15 free-flying OTAs

**OTA Cost  $\sim$  OTA Mass<sup>0.69</sup>** ( $N = 14$ ;  $r^2 = 84\%$ ) *without JWST*

**OTA Cost  $\sim$  OTA Mass<sup>0.72</sup>** ( $N = 15$ ;  $r^2 = 92\%$ ) *with JWST*





# Mass Models

Our data shows that

**Total Mass is ~ 3.3X OTA Mass ( $r^2 = 92\%$ ), and**

**Total Cost is ~3.3X to 5X OTA Cost.**

3.3X comes from WBS analysis

5X comes from regression analysis

<u>Mission</u>	<u>Mass Ratio</u>	<u>Cost Ratio</u>
JWST	~2.6X	~5.3X
Hubble	4.6X	5.5X
Chandra	6.2X	2.8X

For Chandra, science instruments were massive and optics expensive

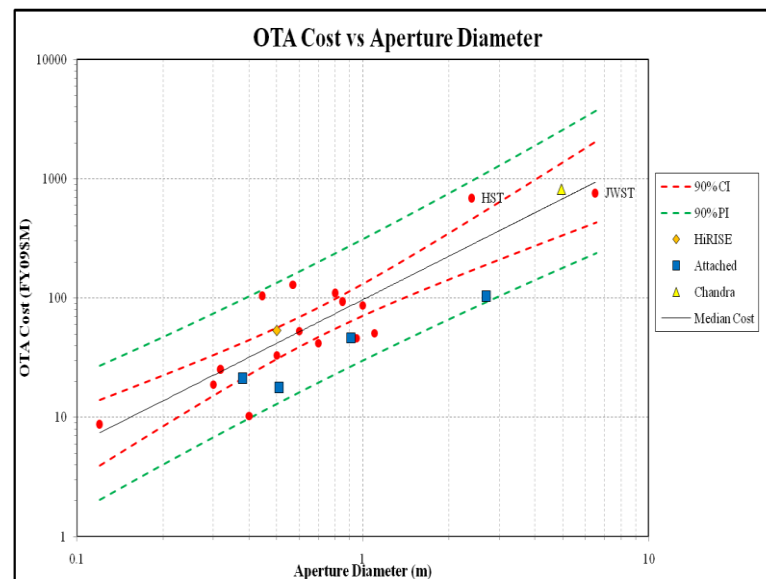
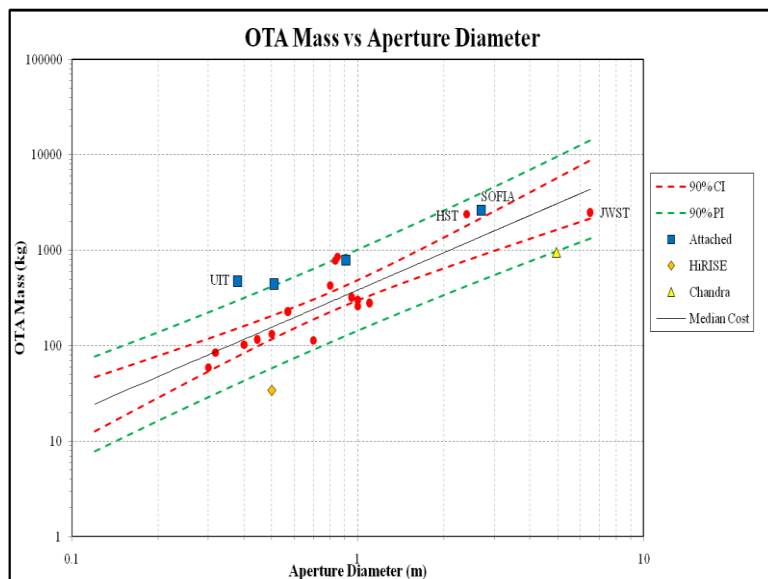


# It costs more to make a Lightweight Telescope

Based on 15 free-flying and 4 attached missions

(3 to Space Shuttle Orbiter and SOFIA to Boeing 747)

For a given Aperture Diameter, OTAs which are ‘attached’ to a large spacecraft (and thus do not have a mass constrained design) tend to be 2X more massive than ‘free-flying’ OTAs and 2.5X less expensive.



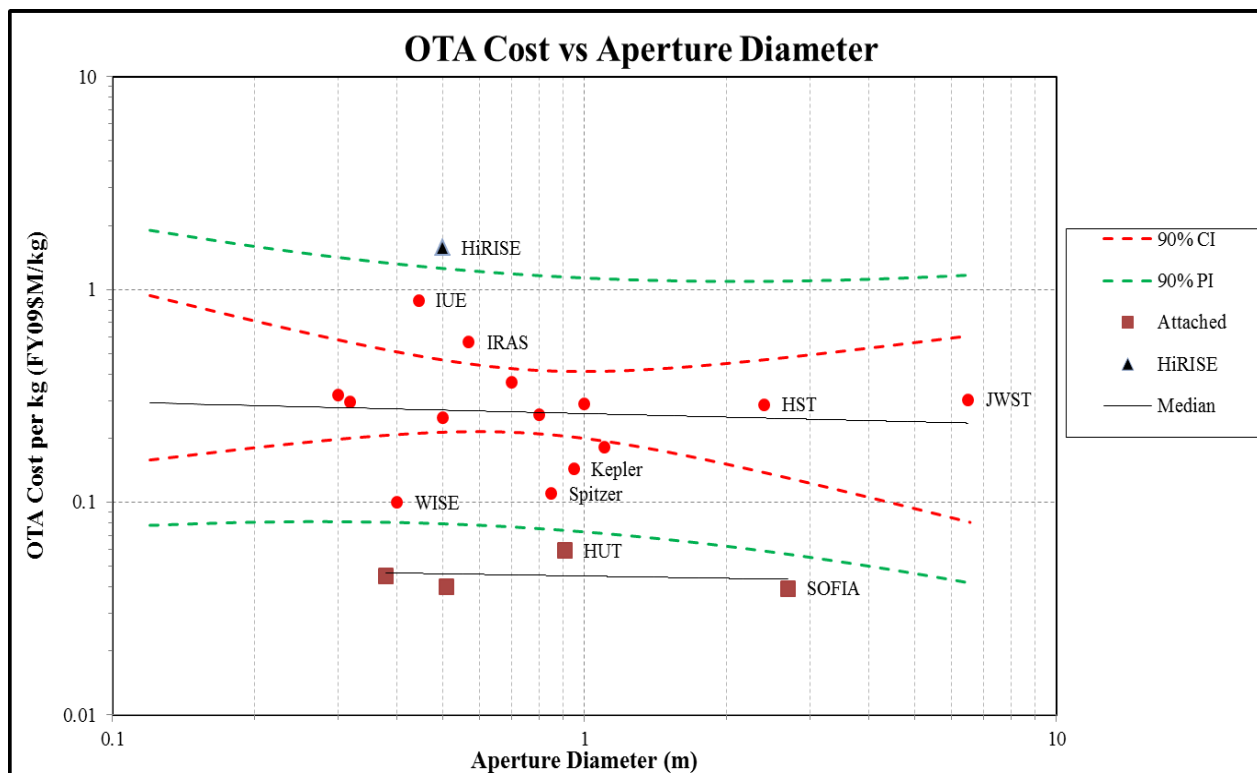


# Space Telescope Mass Constrained Design Classes

Independent of Aperture Diameter, it costs more to design and build a low mass OTA than a high mass OTA.

Free-Flying OTAs are ~5.5X more expensive per kg than Attached OTAs.

There may be a third cost class – Planetary – but we are not certain because HiRISE is our only planetary OTA data point.





## Problem with Mass

Mass may have a high correlation to Cost.

And, Mass may be convenient to quantify.

But, Mass is not an independent variable.

Mass depends upon the size of the telescope.

Bigger telescopes have more mass and Aperture drives size.

And, bigger telescopes typically require bigger spacecraft.

The correlation matrix says that Mass is highly correlated with:

Aperture Diameter, Focal Length,  $F/\#$ , Volume, Pointing and Power

But in reality it is all Aperture, the others all depend on aperture.





# Conclusions



# Conclusions

From engineering & scientific perspective, Aperture Diameter is the best parameter for a space telescope cost model.

But, the single variable model only predicts 75% of OTA Cost:

$$\text{OTA Cost} \sim D^{1.2} \quad (N = 17; r^2 = 75\%; SPE=79\%) \text{ with 2009 JWST}$$

Two Variable Models provide better estimates

$$\text{OTA Cost} \sim D^{1.3} e^{-0.04(\text{LYr}-1960)} \quad (N = 17, r^2 = 93\%; SPE=39\%)$$

$$\text{OTA Cost} \sim D^{1.3} e^{-0.04(\text{YoD}-1960)} \quad (N = 16, r^2 = 95\%; SPE=39\%)$$

A potential Three Variable Model is:

$$\text{OTA Cost} \sim D^{1.15} \lambda^{-0.17} e^{-0.03(\text{YoD}-1960)} \quad (N = 20, r^2 = 92\%; SPE = 76\%)$$

At present the study has not yet produced a satisfactory model for Total Mission Cost.



# Conclusions

OTA mass is not a good CER

OTA mass is multi-collinear with diameter, and  
more massive telescopes actually cost less to make.

For a given aperture diameter, attached OTAs are on average 2X  
more mass and 2.5X less expensive than free-flying OTAs.

Independent of aperture diameter, Cost per kilogram of attached  
OTAs is ~5.5X lower than for free-flying OTAs

There may be a third even more expensive 'design rule' class –  
Planetary OTAs – but we only have one data point.

Bottom line: using Mass as an OTA CER is misleading and could  
easily lead one to make inappropriate programmatic decisions.



## Major Findings

Aperture Diameter is principle cost driver for space telescopes.

Larger diameter telescopes cost less per square meter of collecting aperture than small diameter telescopes.

Technology development reduces cost by 50% per 17 years.

If all other parameters are held constant,  
adding mass reduces cost, and  
reducing mass increases cost.



Backup



# Areal Cost verses Diameter and V2

	coef		p		Areal Cost vs Diameter and V2																	
Second Variable	Diameter alone		PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit		Operating Temperature	
	-0.74	0.00	-1.22	0.06	-0.87	0.01	-1.90	0.22	-0.84	0.03	-0.80	0.00	-1.13	0.10	-0.50	0.10	-0.73	0.00	-0.73	0.04	-0.74	0.00
	-	-	0.34	0.47	0.24	0.59	0.34	0.47	-0.26	0.18	-0.05	0.44	0.31	0.31	0.31	0.31	-0.04	0.61	-0.09	0.59	-0.01	0.93
	52%		40%		37%		40%		42%		51%		15%		15%		48%		18%		48%	
	78%		74%		76%		74%		73%		76%		82%		82%		82%		95%		80%	
n	17		13		13		13		13		16		15		15		17		11		16	
Multicollinearity?	No		Yes		No		Yes		No		No		Yes		No		No		No		No	

Second Variable	Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		YoD (exp)		Development Period		Dev Per (exp)		Launch Date (exp)		Orbit	
Aperture Diameter	-0.59	0.04	-0.60	0.03	-0.73	0.00	-0.80	0.00	-0.69	0.02	-0.68	0.00	-0.76	0.00	-0.76	0.00	-0.61	0.00	-0.72	0.00
Second Variable	-0.15	0.23	-0.08	0.28	-0.02	0.90	0.00	0.60	-0.93	0.02	-0.04	0.00	0.25	0.56	0.00	0.66	-0.04	0.00	0.03	0.53
Adjusted $r^2$	50%		51%		48%		48%		56%		76%		48%		47%		76%		49%	
SPE	78%		59%		80%		80%		35%		39%		74%		76%		40%		83%	
n	16		12		17		17		8		16		16		16		17		15	
Multicollinearity?	No		No		No		No		No		No		No		No		No		No	

TRL, YoD and LYr are all >98% significant. TRL is less noisy, but YoD and LYr have higher correlation values. Getting more TRL data may improve the model.



# Total Mission Cost verses Diameter and V2

	coef		p		Total Cost vs Aperture Diameter and V2																			
Second Variable	Diameter alone		PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		Total Areal Density		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit		Operating Temperature	
Aperture Diameter	0.88	0.01	0.83	0.27	1.25	0.00	0.01	1.00	0.26	0.55	0.75	0.08	1.19	0.05	2.20	0.00	1.34	0.00	0.91	0.01	0.10	0.02	0.88	0.01
Second Variable	-	-	0.41	0.46	0.38	0.75	0.41	0.46	0.11	0.37	-0.08	0.53	0.08	0.83	1.03	0.00	0.08	0.83	-0.08	0.55	-0.23	0.25	-0.04	0.81
Adjusted r <sup>2</sup>	72%		61%		61%		61%		-11%		74%		67%		90%		67%		74%		16%		68%	
SPE	203%		118%		119%		118%		126%		178%		115%		71%		115%		176%		241%		219%	
n	16		12		12		12		13		15		15		15		15		16		10		16	
Multicollinearity?	No		Yes		No		Yes		No		No		Yes		Yes		No		No		No		No	

Second Variable																				
	Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		YoD (exp)		Development Period		Dev Per (exp)		Launch Date (exp)		Orbit	
Aperture Diameter	0.31	0.44	0.55	0.26	0.54	0.13	0.54	0.11	1.77	0.01	0.95	0.01	0.52	0.05	0.36	0.16	0.94	0.01	0.86	0.00
Second Variable	0.37	0.09	0.14	0.36	0.56	0.11	0.01	0.09	-0.52	0.45	-0.03	0.35	2.01	0.00	0.03	0.00	-0.02	0.60	0.16	0.02
Adjusted r <sup>2</sup>	72%		22%		84%		66%		93%		90%		77%		71%		81%		46%	
SPE	124%		219%		138%		155%		112%		197%		97%		96%		209%		174%	
n	15		12		16		16		8		15		15		15		16		14	
Multicollinearity?	No		No		No		No		No		No		No		No		No		No	

As indicated by the very large SPE, there are NO GOOD MODELS for Total Cost vs Diameter or Diameter + V2. Note: Diameter x Areal Density = Mass.

[illegible]



[illegible]



# Total Cost verses Mass and V2

	coef		p	Total Cost vs Total Mass and V2																				
Second Variable	Total Mass alone		PM F Len.		PM F/N		OTA Volume		FOV		Pointing Accuracy		OTA Mass		Total Areal Density		OTA Areal Density		Spectral Range minimum		Wavelength Diffraction Limit		Operating Temperature	
Total Mass	1.11	0.00	1.07	0.02	1.12	0.00	0.97	0.03	1.07	0.00	1.06	0.00	1.20	0.00	1.10	0.00	1.06	0.00	1.14	0.00	1.03	0.00	1.13	0.00
Second Variable	-	-	0.03	0.93	-0.14	0.75	0.06	0.71	0.04	0.61	-0.03	0.60	-0.08	0.69	-0.06	0.54	-0.16	0.47	0.09	0.20	-1.03	0.85	-0.13	0.16
Adjusted r <sup>2</sup>	85%		84%		81%		91%		9%		85%		82%		90%		94%		90%		26%		91%	
SPE	71%		81%		77%		81%		79%		72%		81%		71%		87%		61%		243%		56%	
n	15		11		11		11		12		15		14		15		14		15		9		15	
Multicollinearity?	No		Yes		No		Yes		No		No		Yes		No		No		No		No		No	

Second Variable	Avg. Input Power		Data Rate		Design Life		Design Life (exp)		Technology Readiness Level		YoD (exp)		Development Period		Dev Per (exp)		Launch Date (exp)		Orbit	
Total Mass	1.06	0.00	0.98	0.00	1.04	0.00	1.12	0.00	1.28	0.00	1.12	0.00	0.85	0.00	0.77	0.00	1.10	0.00	1.01	0.00
Second Variable	-0.02	0.92	0.10	0.17	0.12	0.62	0.00	0.96	0.70	0.19	0.01	0.62	1.16	0.01	0.02	0.01	0.01	0.41	0.09	0.01
Adjusted r <sup>2</sup>	94%		95%		81%		85%		96%		90%		95%		98%		93%		96%	
SPE	79%		63%		73%		74%		58%		72%		54%		58%		71%		38%	
n	14		12		15		15		8		15		15		15		15		14	
Multicollinearity?	No		No		No		No		No		No		No		No		No		No	

TRL, Temp and Data Rate are significant at an >80% confidence level. But TRL 'sign' is wrong. Development Period and Orbit are both 99% significant. Neither YoD or LYr are significant.